3차원 Viewing 변환

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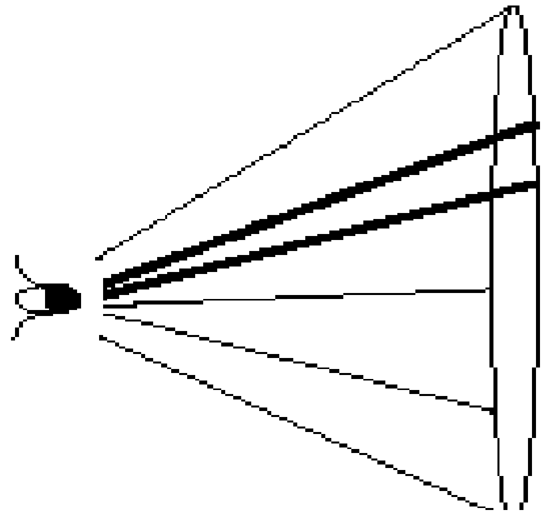
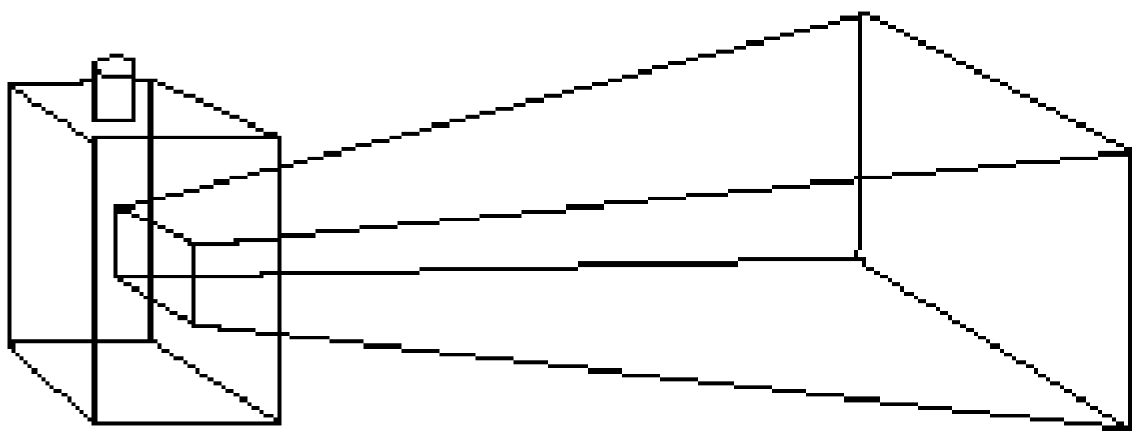
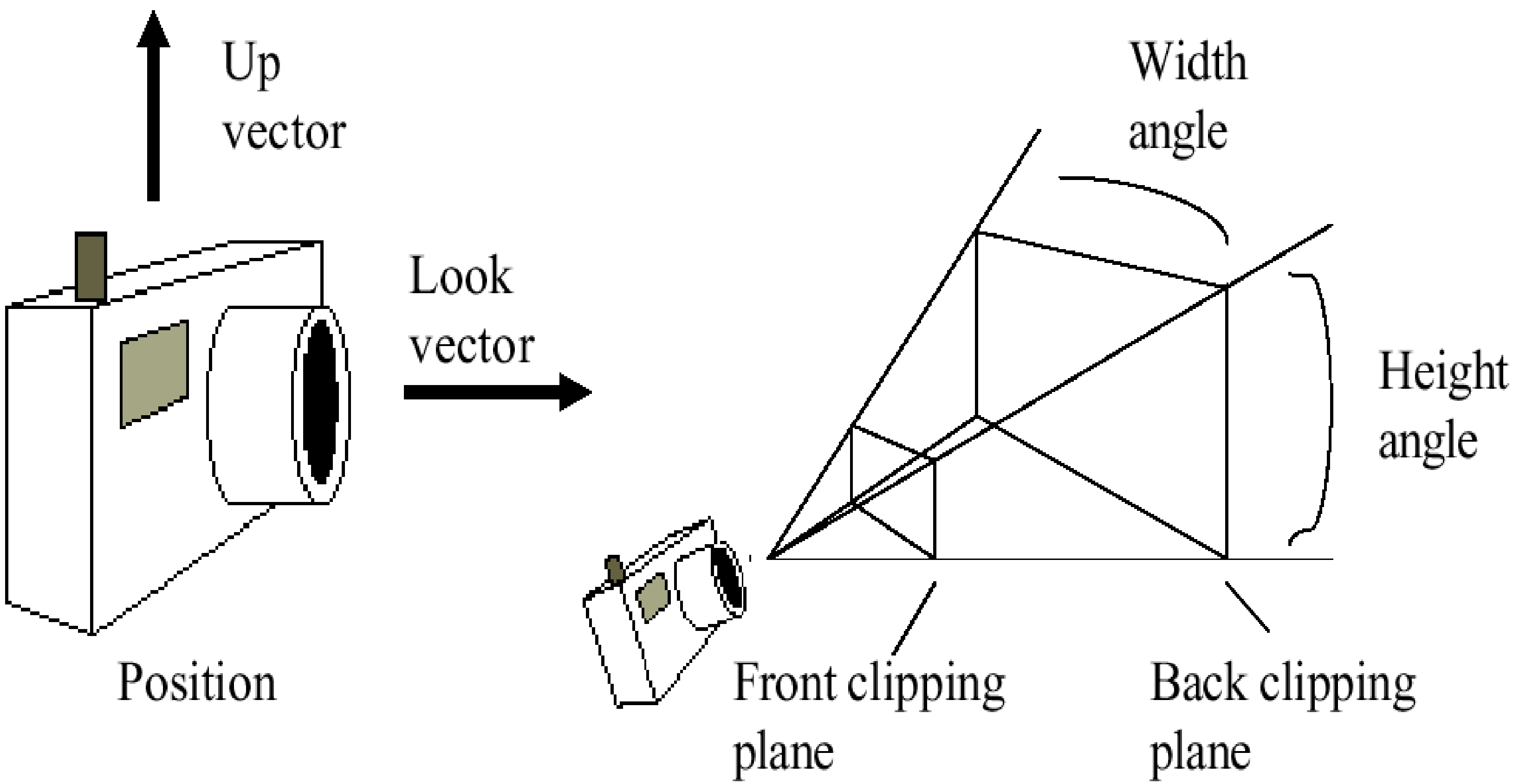
http://cse.snu.ac.kr/mskim
http://3map.snu.ac.kr

3차원 View Volume

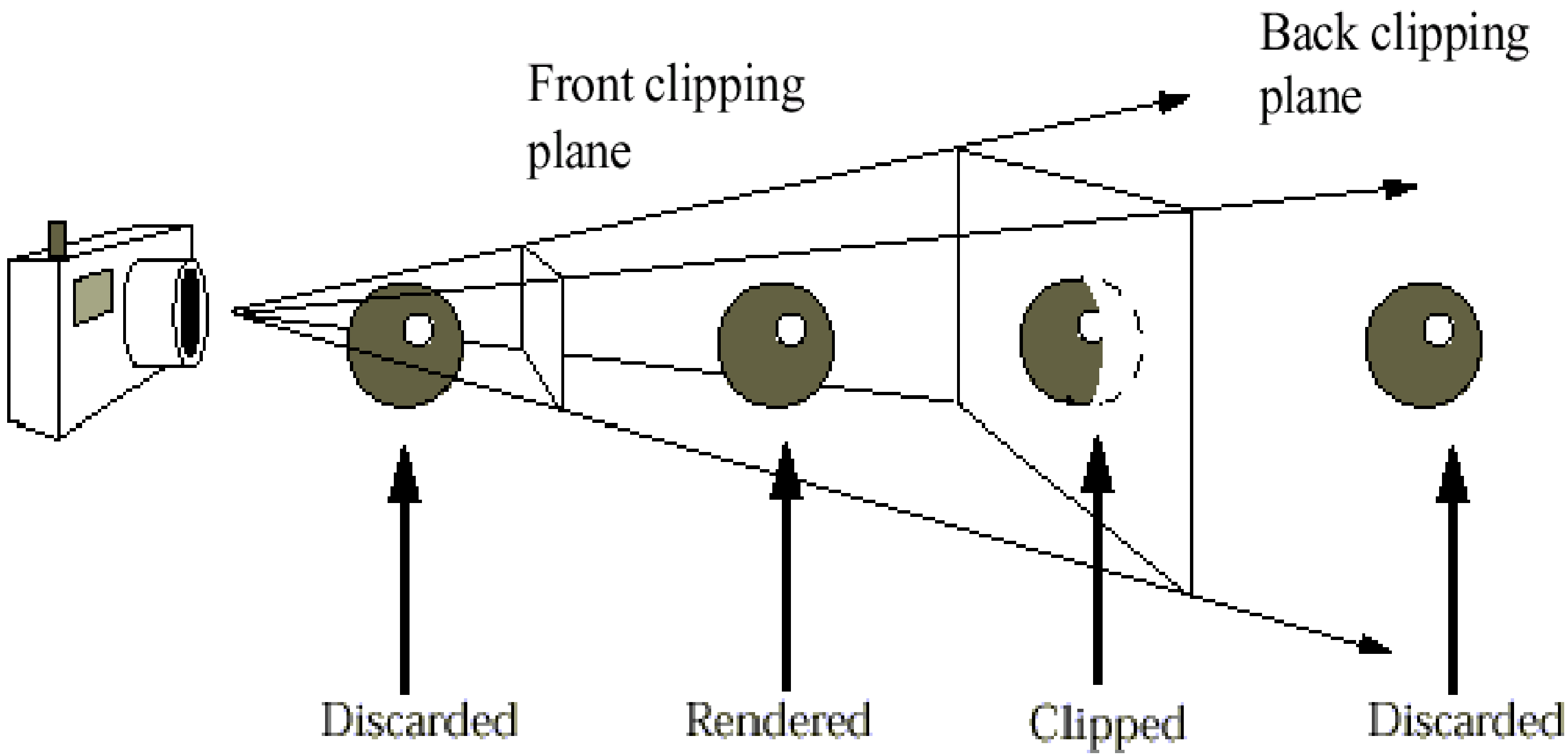




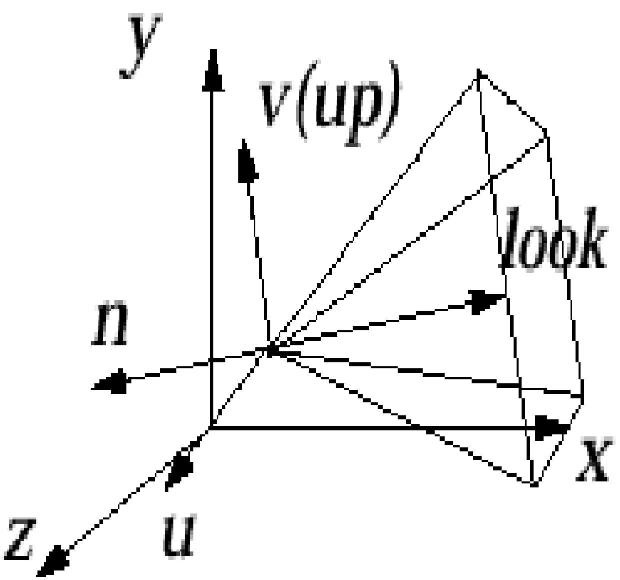
3차원 View Volume



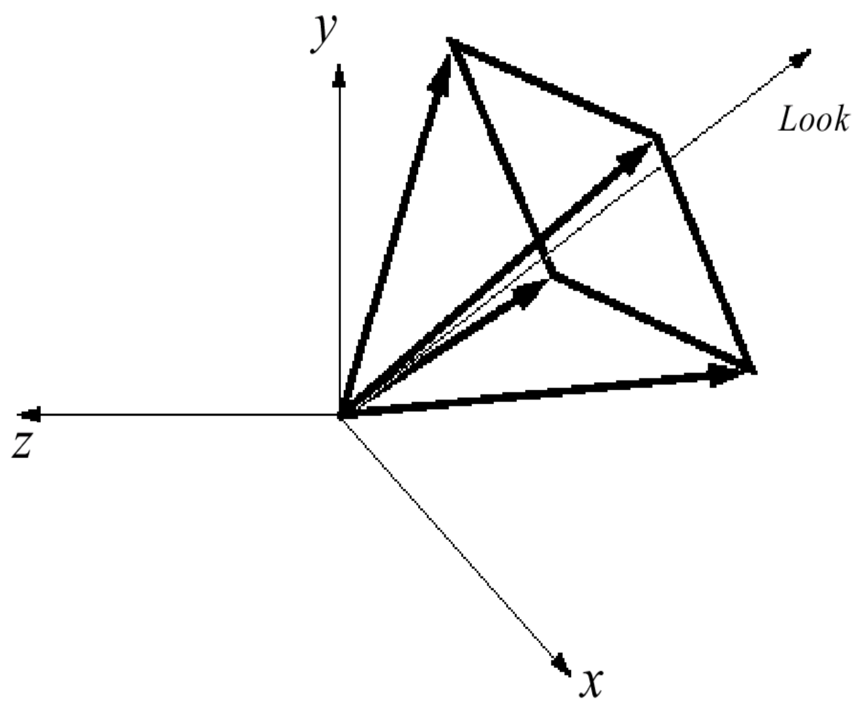
3차원 Clipping



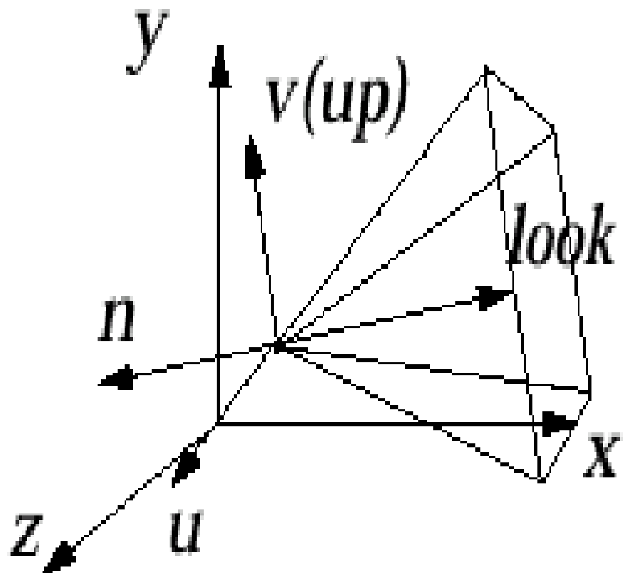
3차원 Viewing 변환



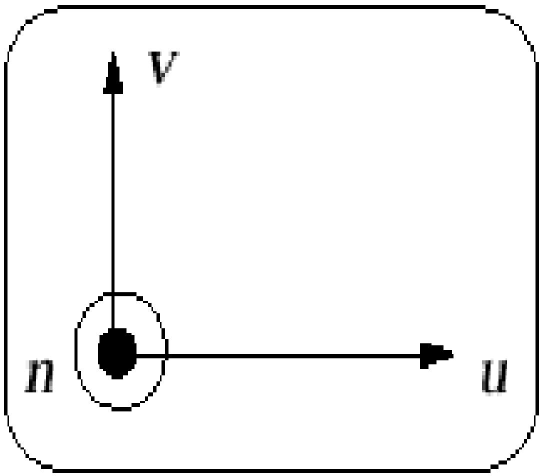
Arbitrary Perspective Frustum



3차원 Viewing 변환



Arbitrary Perspective Frustum



Camera Coordinate System
(with *n* coming out of the paper)

Plane in Space

$$\begin{aligned}
\mathbf{n} &= (a, b, c) \\
\mathbf{x} - \mathbf{x}_0 &= (x - x_0, y - y_0, z - z_0)
\end{aligned}$$

$$\begin{aligned}
0 &= \langle \mathbf{n}, \mathbf{x} - \mathbf{x}_0 \rangle \\
0 &= a(x - x_0) + b(y - y_0) + c(z - z_0) \\
0 &= ax + by + cz - ax_0 - by_0 - cz_0 \\
0 &= ax + by + cz + d
\end{aligned}$$

Plane by Three Points

$$\begin{aligned}\hat{\mathbf{n}} &= (a, b, c, d) \\ \hat{\mathbf{x}}_0 &= (x_0, y_0, z_0, 1) \\ \hat{\mathbf{x}}_1 &= (x_1, y_1, z_1, 1) \\ \hat{\mathbf{x}}_2 &= (x_2, y_2, z_2, 1)\end{aligned}$$

$$\begin{aligned}ax_0 + by_0 + cz_0 + d \cdot 1 &= 0 \\ ax_1 + by_1 + cz_1 + d \cdot 1 &= 0 \\ ax_2 + by_2 + cz_2 + d \cdot 1 &= 0\end{aligned}$$

Plane by Three Points

$$\begin{aligned} ax_0 + by_0 + cz_0 + d \cdot 1 &= 0 \\ ax_1 + by_1 + cz_1 + d \cdot 1 &= 0 \\ ax_2 + by_2 + cz_2 + d \cdot 1 &= 0 \end{aligned}$$

$$\begin{aligned} \langle \widehat{\mathbf{n}}, \widehat{\mathbf{x}}_0 \rangle &= 0 \\ \langle \widehat{\mathbf{n}}, \widehat{\mathbf{x}}_1 \rangle &= 0 \\ \langle \widehat{\mathbf{n}}, \widehat{\mathbf{x}}_2 \rangle &= 0 \\ \widehat{\mathbf{n}} &= \widehat{\mathbf{x}}_0 \wedge \widehat{\mathbf{x}}_1 \wedge \widehat{\mathbf{x}}_2 \end{aligned}$$

Wedge Product

$$\hat{\mathbf{n}} = \hat{\mathbf{x}}_0 \wedge \hat{\mathbf{x}}_1 \wedge \hat{\mathbf{x}}_2$$

$$= \begin{vmatrix} \mathbf{e}_1 & \mathbf{e}_2 & \mathbf{e}_3 & \mathbf{e}_4 \\ x_0 & y_0 & z_0 & 1 \\ x_1 & y_1 & z_1 & 1 \\ x_2 & y_2 & z_2 & 1 \end{vmatrix}$$

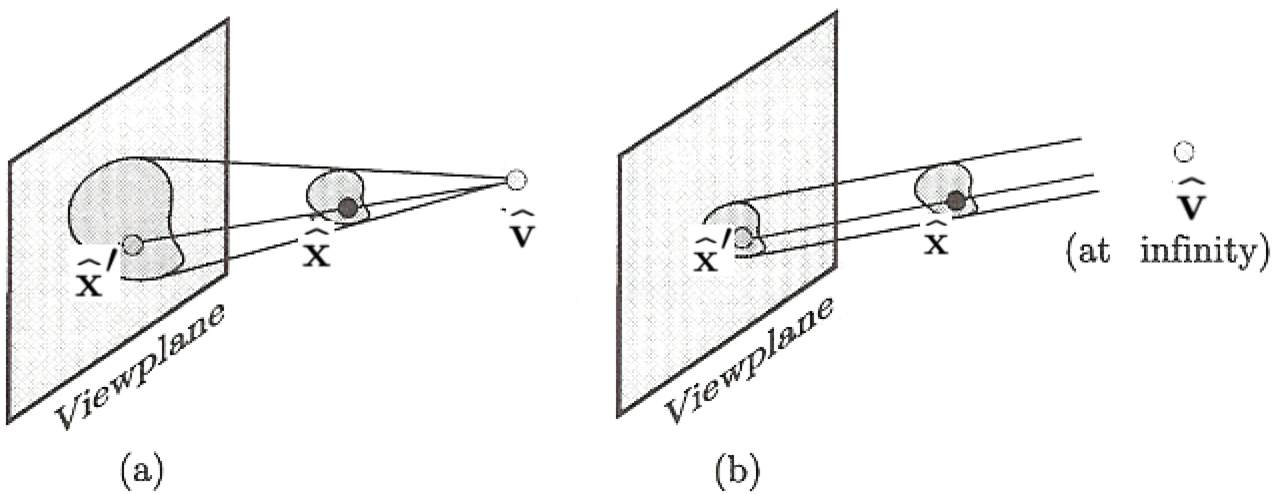
Point by Three Planes

$$
\begin{aligned}
\widehat{\mathbf{x}} &= (x, y, z, w) \\
\widehat{\mathbf{n}}_0 &= (a_0, b_0, c_0, d_0) \\
\widehat{\mathbf{n}}_1 &= (a_1, b_1, c_1, d_1) \\
\widehat{\mathbf{n}}_2 &= (a_2, b_2, c_2, d_2)
\end{aligned}
$$

$$\widehat{\mathbf{x}} = \widehat{\mathbf{n}}_0 \wedge \widehat{\mathbf{n}}_1 \wedge \widehat{\mathbf{n}}_2$$

Projection in 3D

$$\widehat{\mathbf{x}}' = \langle \widehat{\mathbf{n}}, \widehat{\mathbf{v}} \rangle \, \widehat{\mathbf{x}} - \widehat{\mathbf{v}} \, \langle \widehat{\mathbf{n}}, \widehat{\mathbf{x}} \rangle$$



Perspective and parallel three-dimensional projections

Projection in 3D

$$\widehat{\mathbf{x}}' \;=\; \langle \widehat{\mathbf{n}}, \widehat{\mathbf{v}} \rangle \, \widehat{\mathbf{x}} - \widehat{\mathbf{v}} \, \langle \widehat{\mathbf{n}}, \widehat{\mathbf{x}} \rangle$$

(Case I): $\langle \hat{\mathbf{n}}, \hat{\mathbf{x}} \rangle = 0$,

$$\widehat{\mathbf{x}}' = \langle \widehat{\mathbf{n}}, \widehat{\mathbf{v}} \rangle \, \widehat{\mathbf{x}} = \widehat{\mathbf{x}}$$

(Case II): $\langle \hat{\mathbf{n}}, \hat{\mathbf{x}} \rangle \neq 0$,

$$\begin{aligned} \widehat{\mathbf{x}}' &= \alpha \hat{\mathbf{x}} + \beta \hat{\mathbf{v}} \\ 0 &= \left\langle \hat{\mathbf{n}}, \hat{\mathbf{x}}' \right\rangle \\ 0 &= \langle \hat{\mathbf{n}}, \alpha \hat{\mathbf{x}} + \beta \hat{\mathbf{v}} \rangle \end{aligned}$$

Projection in 3D

$$\widehat{\mathbf{x}}' = \langle \widehat{\mathbf{n}}, \widehat{\mathbf{v}} \rangle \widehat{\mathbf{x}} - \widehat{\mathbf{v}} \langle \widehat{\mathbf{n}}, \widehat{\mathbf{x}} \rangle$$

(Case II): $\langle \hat{\mathbf{n}}, \hat{\mathbf{x}} \rangle \neq 0$,

$$\begin{aligned}
\hat{\mathbf{x}}' &= \alpha \hat{\mathbf{x}} + \beta \hat{\mathbf{v}} \\
0 &= \langle \hat{\mathbf{n}}, \hat{\mathbf{x}}' \rangle \\
0 &= \langle \hat{\mathbf{n}}, \alpha \hat{\mathbf{x}} + \beta \hat{\mathbf{v}} \rangle \\
0 &= \alpha \langle \hat{\mathbf{n}}, \hat{\mathbf{x}} \rangle + \beta \langle \hat{\mathbf{n}}, \hat{\mathbf{v}} \rangle \\
\alpha &= -\beta \frac{\langle \hat{\mathbf{n}}, \hat{\mathbf{v}} \rangle}{\langle \hat{\mathbf{n}}, \hat{\mathbf{x}} \rangle}
\end{aligned}$$

Projection in 3D

$$\widehat{\mathbf{x}}' = \langle \widehat{\mathbf{n}}, \widehat{\mathbf{v}} \rangle \, \widehat{\mathbf{x}} - \widehat{\mathbf{v}} \, \langle \widehat{\mathbf{n}}, \widehat{\mathbf{x}} \rangle$$

(Case II): $\langle \widehat{\mathbf{n}}, \widehat{\mathbf{x}} \rangle \neq 0$,

$$\alpha = -\beta \frac{\langle \widehat{\mathbf{n}}, \widehat{\mathbf{v}} \rangle}{\langle \widehat{\mathbf{n}}, \widehat{\mathbf{x}} \rangle}$$

$$\begin{aligned}
\widehat{\mathbf{x}}' &= -\beta \frac{\langle \widehat{\mathbf{n}}, \widehat{\mathbf{v}} \rangle}{\langle \widehat{\mathbf{n}}, \widehat{\mathbf{x}} \rangle} \widehat{\mathbf{x}} + \beta \widehat{\mathbf{v}} \\
&= \frac{\langle \widehat{\mathbf{n}}, \widehat{\mathbf{v}} \rangle}{\langle \widehat{\mathbf{n}}, \widehat{\mathbf{x}} \rangle} \widehat{\mathbf{x}} - \widehat{\mathbf{v}} \\
&= \langle \widehat{\mathbf{n}}, \widehat{\mathbf{v}} \rangle \, \widehat{\mathbf{x}} - \langle \widehat{\mathbf{n}}, \widehat{\mathbf{x}} \rangle \, \widehat{\mathbf{v}}
\end{aligned}$$

Perspective Projection in 3D

$$\widehat{\mathbf{x}}' = \langle \widehat{\mathbf{n}}, \widehat{\mathbf{v}} \rangle \, \widehat{\mathbf{x}} - \widehat{\mathbf{v}} \, \langle \widehat{\mathbf{n}}, \widehat{\mathbf{x}} \rangle$$

$$\begin{bmatrix} x'w' \\ y'w' \\ z'w' \\ w' \end{bmatrix} = \begin{bmatrix} a & b & c & d \end{bmatrix} \begin{bmatrix} v_x \\ v_y \\ v_z \\ 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix}$$

$$- \begin{bmatrix} v_x \\ v_y \\ v_z \\ 1 \end{bmatrix} \begin{bmatrix} a & b & c & d \end{bmatrix} \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix}$$

Parallel Projection in 3D

$$\widehat{\mathbf{x}}' = \langle \widehat{\mathbf{n}}, \widehat{\mathbf{v}} \rangle \, \widehat{\mathbf{x}} - \widehat{\mathbf{v}} \, \langle \widehat{\mathbf{n}}, \widehat{\mathbf{x}} \rangle$$

$$\begin{bmatrix} x'w' \\ y'w' \\ z'w' \\ w' \end{bmatrix} = \begin{bmatrix} a & b & c & d \end{bmatrix} \begin{bmatrix} v_x \\ v_y \\ v_z \\ 0 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix}$$

$$- \begin{bmatrix} v_x \\ v_y \\ v_z \\ 0 \end{bmatrix} \begin{bmatrix} a & b & c & d \end{bmatrix} \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix}$$

Viewing Transform in 3D

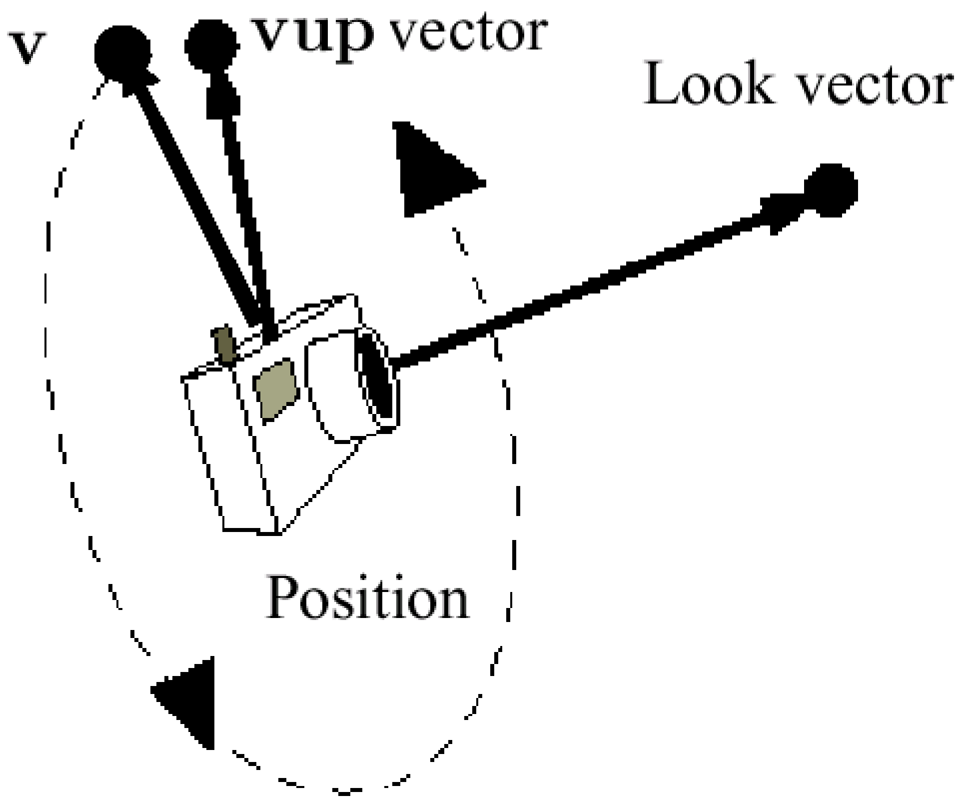
$$\begin{aligned} \mathbf{x} &= (x_0, y_0, z_0) \\ \mathbf{u} &= (u_1, u_2, u_3) \\ \mathbf{v} &= (v_1, v_2, v_3) \\ \mathbf{n} &= (n_1, n_2, n_3) \end{aligned}$$

$$\langle \mathbf{u}, \mathbf{v} \rangle = \langle \mathbf{u}, \mathbf{n} \rangle = \langle \mathbf{v}, \mathbf{n} \rangle = 0$$

$$\|\mathbf{u}\| = \|\mathbf{v}\| = \|\mathbf{n}\| = 1$$

$$\begin{bmatrix} u_1 & u_2 & u_3 & 0 \\ v_1 & v_2 & v_3 & 0 \\ n_1 & n_2 & n_3 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & -x_0 \\ 0 & 1 & 0 & -y_0 \\ 0 & 0 & 1 & -z_0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

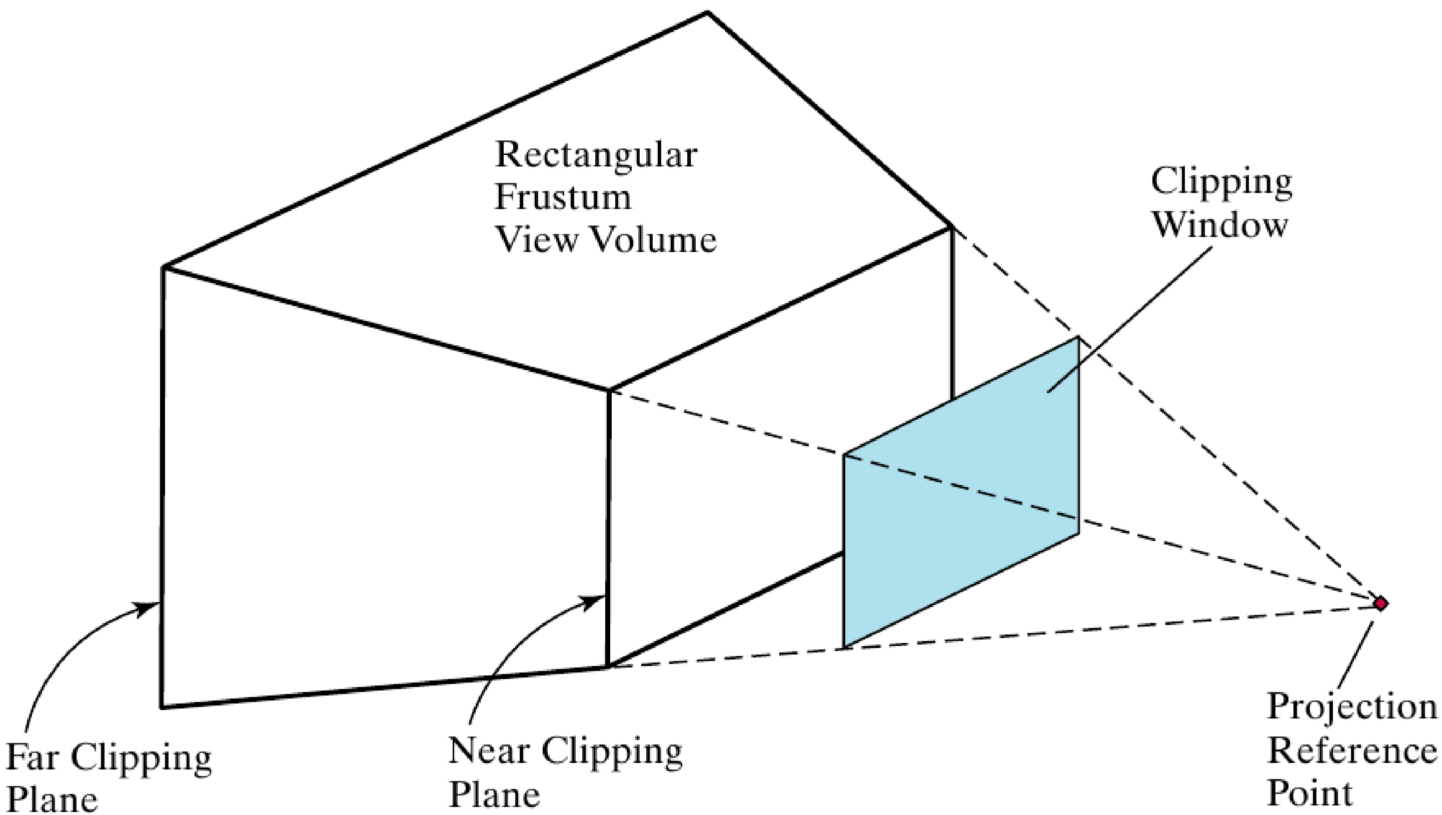
3차원에서의 Viewing 변환



$$\mathbf{u} = \frac{\mathbf{vup} \times \mathbf{n}}{\|\mathbf{vup} \times \mathbf{n}\|}$$

$$\mathbf{v} = \mathbf{n} \times \mathbf{u}$$

Depth Transform in 3D



Depth Transform in 3D

$$ax + by + cz + d_0 = 0$$ Near Clipping Plane

$$ax + by + cz + d_1 = 0$$ Far Clipping Plane

$$ax + by + cz + d = 0$$

How can you check whether the given point (x,y,z) is between the two planes?

How can you formulate the relative depth as an affine transformation?

Depth Transform in 3D

$$ax + by + cz + d_0 = 0$$ Near Clipping Plane

$$ax + by + cz + d_1 = 0$$ Far Clipping Plane

$$ax + by + cz + d = 0$$

3차원에서의 Viewing 변환

- 3차원 상의 점들을 2차원 viewing 평면으로 투영
- Viewing 평면상의 기준점 (x0,y0,z0)을 원점으로 이동
- U, V, N 방향을 X, Y, Z의 좌표계의 방향으로 회전
- 결과적으로 XY 평면상에 투영된 그림이 나타난다.